CSE509: (Intro to) Systems Security

Fall 2012

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Key Exchange
Public Key Cryptography

Public Key Cryptography

- Fundamentals
- RSA

Parenthesis: Key Exchange

- Compute a common, shared key
  - Called a *symmetric key exchange protocol*
- Challenges:
  - I don’t know the other party
  - Alice and Bob vs. Eve (who eavesdroppes)

Key Exchange: Idea

- Alice: generates random **a**
- Bob: generates random **b**
- Alice sends: $\mathbf{m_a = g^a}$
- Bob sends: $\mathbf{m_b = g^b}$
- Alice does: $\mathbf{(m_b)^a} = \mathbf{g^{ba} = key}$
- Bob does: $\mathbf{(m_a)^b} = \mathbf{g^{ab} = key}$
- Does it work ?!!! Seems very simple !

Diffie-Hellman

- Discrete logarithm problem hardness:
  - Given integers n and g and prime number p, compute k such that $n = g^k \bmod p$
  - Solutions known for small p
  - Solutions computationally infeasible as p grows large

Algorithm

- Constants: prime p, integer $g \neq 0, 1, p-1$
  - Known to all participants
- Alice chooses private key k_{Alice}, computes public key $K_{Alice} = g^{k_{Alice}} \bmod p$
- To communicate with Bob, Alice computes

  $K_{shared} = K_{Bob}^{k_{Alice}} \bmod p$
- To communicate with Alice, Bob computes

  $K_{shared} = K_{Alice}^{k_{Bob}} \bmod p$
  - It can be shown these keys are equal

Problems

- Man in The Middle (MITM)
  - solution: authenticate first
- Are we talking to the right person ?
- Forward Secrecy (PFS)
  - future compromise does not impact past
  - station to station (STS) Protocol

Overview

$$M = D_{private_B}(E_{public_B}(M))$$

How does Alice know whom it talks to ?
What if Mallory simply replaces the public key with something else (e.g., own) !

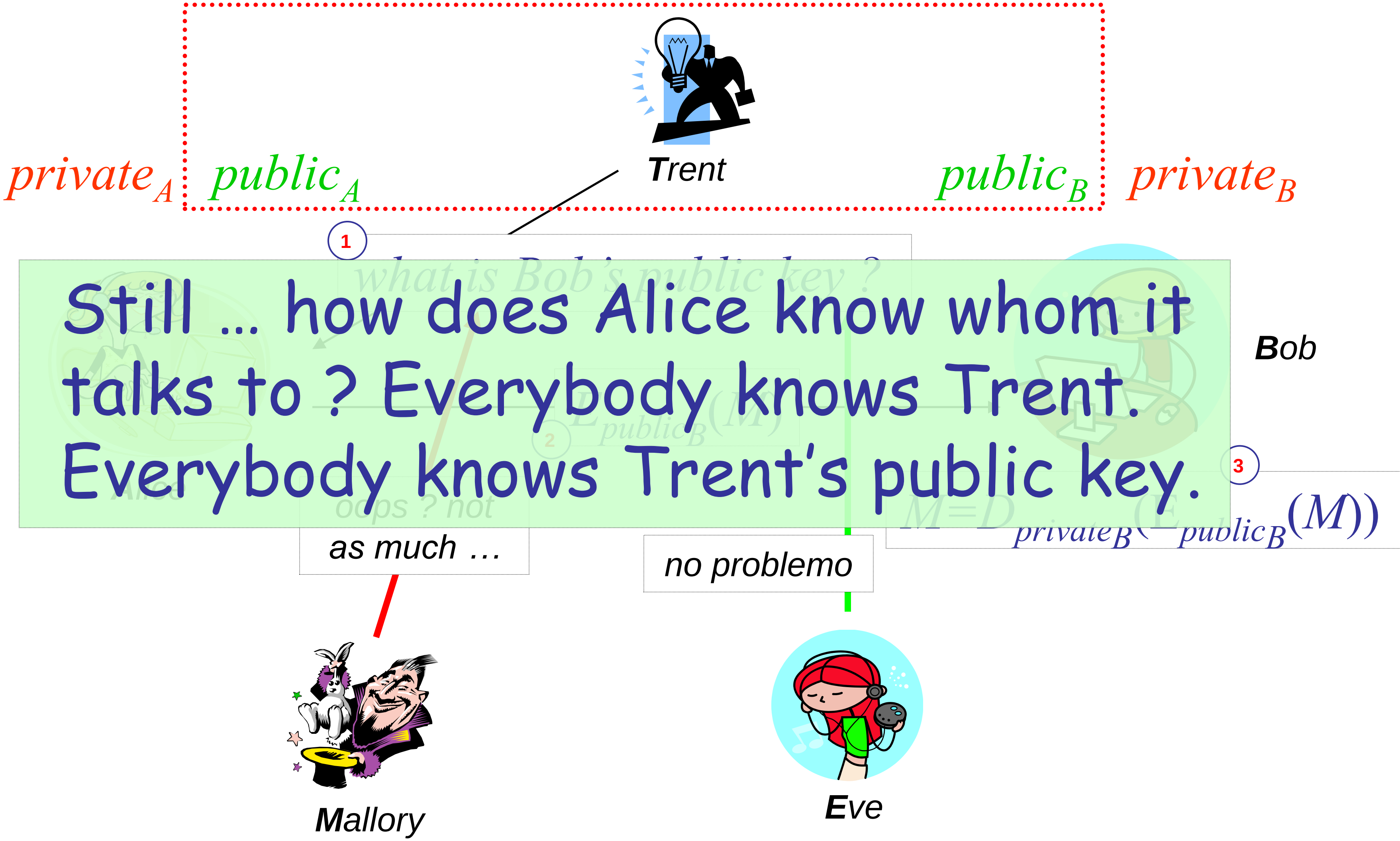
Enter signatures …

Signature …

… something that **only signer can produce**
… and **everybody can verify**

verify = check for a unique association between the signer identity, text to be “signed” and the signature.

Certificate Authority



Still … how does Alice know whom it talks to ? Everybody knows Trent. Everybody knows Trent's public key.

Security services we can now offer

- Confidentiality
  - Only the owner of the private key knows it, so text enciphered with public key cannot be read by anyone except the owner of the private key
- Authentication
  - Only the owner of the private key knows it, so text enciphered with private key must have been generated by the owner (“digital signature”)
    - In real life: encrypt a hash of the text only !!!

More Security Services

- Integrity
  - Enciphered letters cannot be changed undetectably without knowing private key
- Non-Repudiation
  - Message enciphered with private key came from someone who knew it

Public-key crypto: some of the requirements

1. It must be computationally easy to encipher or decipher a message given the appropriate key
2. It must be computationally infeasible to derive the private key from the public key
3. It must be computationally infeasible to determine the private key from a chosen plaintext attack

How are we going to pull it off ?

Trapdoor function (Diffie and Hellman 1976): function that is easy to compute but <u>believed</u> hard to invert without additional information (the "trapdoor"). We can then make the trapdoor the secret key ☺

Example: factoring primes (computing $n=p*q$ is easy, but given n, finding p and q is <u>believed</u> to be hard)

Things <u>can be proven otherwise after a while</u>: e.g., Merkle-Hellman Knapsack cryptosystem

Not all hard problems are trapdoors: e.g., discrete logarithm problem-related functions

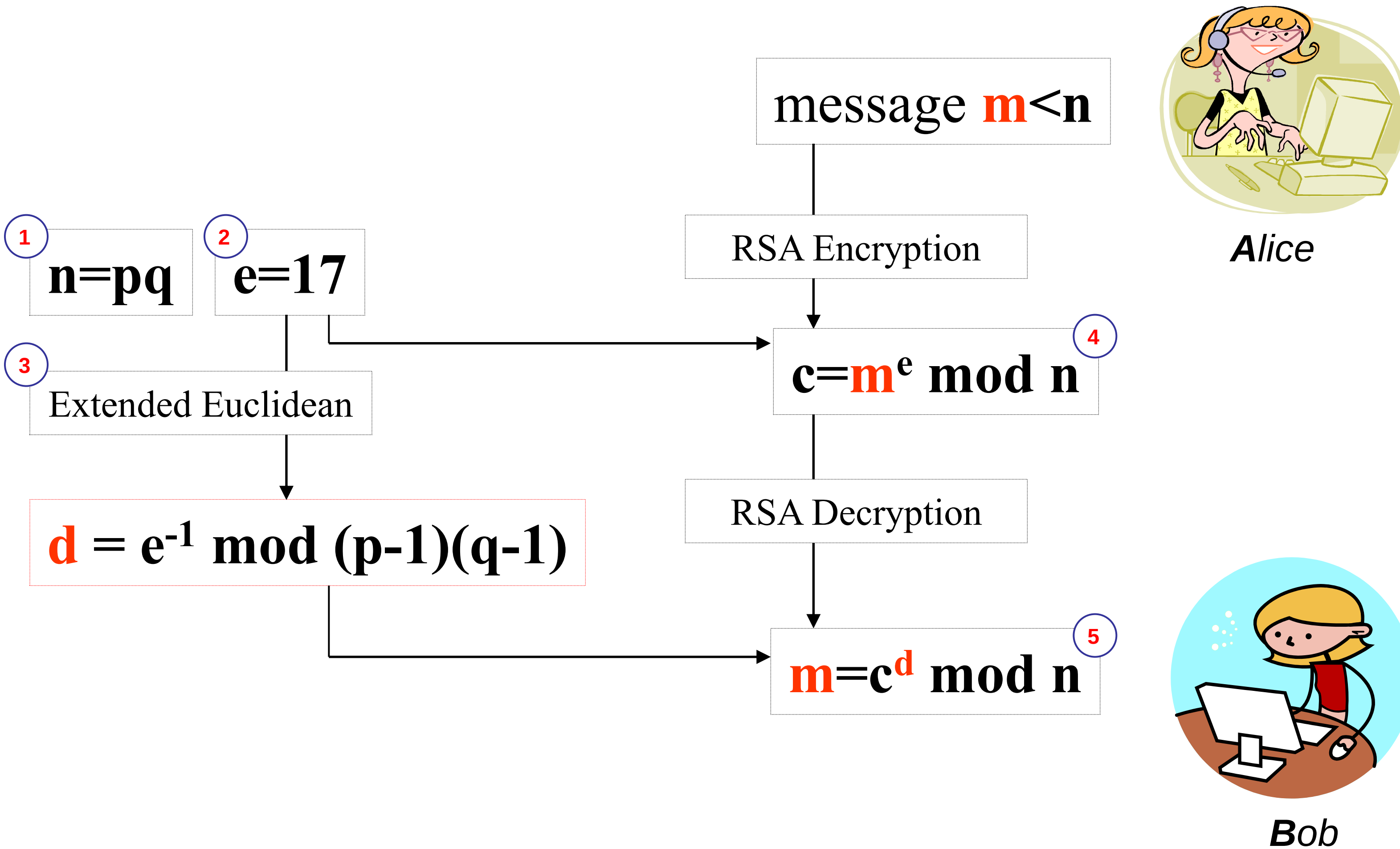
RSA: Rivest,Shamir,Adleman

- Exponentiation cipher
- Relies on the difficulty of determining the number of numbers relatively prime to a large integer n
- Or equivalently, on the difficulty of factoring of large numbers into prime factors

Algorithm

- Key generation
  - Choose large primes p,q; let **n=pq**
  - Choose e relatively prime to (p-1)(q-1) (to have inverse !)
  - Public key **<e,n>**
  - Private key **<d,n>** where **d = e^{-1} mod (p-1)(q-1)**
    - Extended Euclidean (see book Chapter 11)
- Encrypt: **c = m^e mod n**
- Decrypt: **m = c^d mod n**
- de = 1 mod (p-1)(q-1), so m = (me)d mod n
- Breakable if we can factor ☺

RSA Algorithm (animated version)



What about larger messages ?

- Break message into pieces no greater in value than n-1 (why ?)
- Encrypt each part separately
- Use some sort of "chaining" to avoid block-related attacks
- Will likely use some padding etc. We discuss this later.

Vulnerabilities

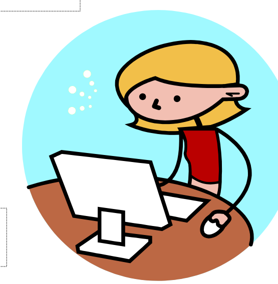
- Attack: Exhaustive search for key
- Attack: Factoring n
- Timing Attacks: how long does encryption take ? – leaks information about the key
  - Solutions ?
- Attack: maintain dictionary of encrypted (public key) messages ("forward search")
- Common modulus problem
- etc. (many solved using smart padding)

RSA Common Modulus Problem Illustration

same modulus **n**

later, same message **m**

Alice



Bob

(1) $c_1 = m^{e_1} \bmod n$

(1) $c_2 = m^{e_2} \bmod n$

public network

Eve

Modified Extended Euclidean

(2) find r, s: $re_1 + se_2 = 1$

Extended Euclidean

(3) $c_2^{-1} \bmod n$

(4) $m = c_1^{r} * (c_2^{-1})^{-s} = m^{re_1 + se_2} \bmod n$

More Problems

- Malleable (public key is known!)
- Probing
  - If I get e(m), I can check if m=m’
  - Solution: random pad – we discuss semantic security later
- Efficiency: can be made faster (modulo calculus tricks)
- Potential use interference: Encryption with Signatures
- Generating keys expensive
  - Select large primes
  - Find e relatively prime to (p-1)(q-1)
    - In practice, often e=3,5,17,65537
- For x<n no modular reduction takes place !!!
  - Also, given a signatures for m1, m2; can compute signature for (some) other messages

Predictors for least significant bits of RSA plaintext

- RSA “reveals” last bit of message
  - The correct reasoning for this is more complicated and was made in a series of papers, such as:
    - Håstad, J. and Näslund, M. 1998. The Security of Individual RSA Bits. In *Proceedings of the 39th Annual Symposium on Foundations of Computer Science* (November 08 - 11, 1998). FOCS. IEEE Computer Society, Washington, DC, 510.
    - S. Goldwasser, S. Micali and P. Tong, *Why and how to establish a private code on a public network*, FOCS'82, pp. 134-144.
    - M. Ben-Or, B. Chor, and A. Shamir. *On the cryptographic security of single RSA bits*. In Proc. 15th ACM Symposium on Theory of Computing, pages 421-430, ACM, Boston, 1983
    - Chor, B. and Goldreich, O., "*RSA/Rabin Least Significant Bits Are 1/2+1/poly(log n) Secure*", in Advances in Cryptology: Proc. of Crypto 84, G. R. Blakley and D. Chaum, eds., Lecture Notes in Computer Science 196, Springer-Verlag, Berlin, 1985, pp.303-313.
    - R. Fischlin and C.P. Schnorr, “*Stronger security proofs for RSA and Rabin bits”,* Journal of Cryptology, 13 (2), pp.221-244, 2000.

Close Parenthesis: back to DH

- Man in the middle solution: authentication and signatures on certain messages by first acquiring public/private key pairs
  - But why not use these keys to communicate then (instead of generating key every time) ?
    - Perfect forward secrecy ☺

Think about this

- Which one goes first:
  - Authentication or Key Exchange ?